A Preliminary Trajectory Design to Uranus and Varuna

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Abstract

A preliminary design is created for a solar system escape trajectory that passes by Uranus and the trans-Neptunian object Varuna (2020000). In this mission, the spacecraft departs Earth in late 2029 with an energy (C3) of 15.79 km^2/s^2 allowing for up to 11,000 kg of mass to be delivered on an expendable SpaceX Falcon Heavy. The spacecraft utilizes Venus and two Earth gravity assists to increase its heliocentric energy. The sequence continues to the Jupiter flyby which places it on a solar system escape trajectory. A final Uranus flyby is included to target the highly inclined orbit of Varuna, and encounters the TNO within 17 years from launch. The mission requires 290 m/s of trajectory correction maneuver ΔV, but is able to send a significantly large payload on a solar system escape of 4.3 AU/year making it a fast and novel mission design to study Uranus, Varuna, and the interstellar medium.

1 Introduction

Missions to the outer reaches of our solar system are few and far between. In most cases, trajectories for these missions require high launch energies for direct Earth to Jupiter transfers and thus deliver small probes to the ice giants and beyond. Currently Pioneer 10/11, Voyager I/II, and more recently New Horizons are the only probes to escape the solar system - relaying back valuable information of the gas giants, Kuiper Belt Objects (KBOs) and Trans-Neptunian Objects (TNOs). However, there is still much to learn about this relatively unexplored region of our cosmic neighborhood. The most recent Planetary Science Decadal Survey, published in April 2022, marked a flagship mission to Uranus as being a key objective for NASA.[1] With such interest to study the ice giants of our solar system, and the other-worldy asteroids that lay beyond the orbit of Jupiter, a flagship caliber mission to accomplish these goals is of upmost importance.

Varuna (2020000) TNO Orbital Elements JD2459600.5 (2022-Jan-21.0) TDB		
Semi-Major Axis	(au)	42.76176312532175
Eccentricity	-	0.05780618206824055
Inclination	$(deg.)$	17.19783739539992
RAAN	$(deg.)$	97.32981964113675
Argument of Periapsis	$(deg.)$	264.3848813162708
Mean Anomaly	$(deg.)$	118.8175601898251
Period	$(years)$	279.6350510603211
Date of Last Periapsis	-	1929-Oct-05.90615284

Table 1: Varuna Orbital Elements from SSD Small-Body Database Lookup.

In this preliminary mission design, a trajectory to Uranus and to the TNO Varuna is considered. Varuna has the designation 2020000 in the Jet Propulsion Laboratory (JPL) Solar System Dynamics (SSD) Horizons interface. This is a web-based or downloadable ephemeris solution. Varuna's orbital

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[1] Origins, Worlds, and Life. A Decadal Strategy for Planetary Science and Astrobiology 2023-2032: https://nap.nationalacademies.org/catalog/26522/origins-worlds-and-life-a-decadal-strategy-for-planetary-science.

elements at January 21, 2022 TDB are pulled from the Small-Body Database Lookup and are presented in Table 1.[2] Varuna was chosen for its apparent diameter of 900 km ±140 km and its absolute magnitude of 3.79. This relatively large TNO is accessible via a Jupiter or Jupiter and Uranus gravity assist in the mission design time restrictions.

This report discusses the computation methods and software used to find the trajectory, the preliminary design process, its implementation in a higher fidelity model, and finally discussion on the trajectory characteristics. Section 2 consists of the tools developed and used, Section 3 is the sequence search and optimization, and Section 4 outlines the higher fidelity GMAT trajectory model.

2 Mission Design Tools

Designing this trajectory began with a literature search for flyby trajectories to KBOs and TNOs[3]. Several papers have been published regarding solar system escape itineraries, but these commonly start with Jovian gravity assists with large incoming V_∞ magnitudes making a direct transfer from Earth infeasible. Therefore, a trajectory connecting the TNO encounter to Jupiter, either directly or via another outer planetary flyby, is used as a starting point for the trajectory design process. The trajectory can then be solved backwards by connecting an inner planet flyby to Jupiter with a large departure V_∞ magnitude. Several flybys of the inner planets need to be conducted to bring the launch energy down, and thus increase mass delivered. The process to find this sequence, and to connect it to the escape trajectory is discussed sequentially in the following tools.

2.1 MATLAB Codes

2.1.1 Broad Trajectory Search

Before an ideal sequence is found, a coarse search of the solution space is required to find possible flyby combinations. Useful tools such as Porkchop or Tisserand plots can help to show where transfers can occur and if they're possible from an energy standpoint. However, as the number of encounters increases, the complexity of keeping track of each encounter via plots becomes challenging and time consuming. Therefore, we can employ a broad search algorithm to coarsely search encounter dates and trajectories between bodies that can later be refined. The broad search algorithm begins by evenly spacing encounter Julian dates between an upper and lower bound per planetary encounter. Lambert arcs connect every single encounter date between bodies in a sequence until a set of arcs from the departure to the arrival body are created. Because the dates are fixed at each encounter, a trajectory can be matched from the starting body and can pass through the intermediate body with no time difference (the instantaneous flyby assumption), and onto the next body in the sequence. Figure 1 illustrates this method connecting the Earth departure (E) with the second Earth encounter via a Venus gravity assist (V). In the example, four Julian dates are created at each encounter, and so the total number of resulting Lambert arcs to compute is 32 to connect each time between each encounter. It's important to note that not every date pair will result in a successful Lambert solution, and so the leg is discarded. The incoming and outgoing V_∞ magnitude difference acts as a "cost" to evaluate the trajectories. The associated B-Plane components and radius of close approach can also be reasonably computed with the V_∞ vectors. Filtering criteria are established based on the flyby altitude and cost to connect each transfer. Finally, this process is done recursively for each encounter until the beginning and ending bodies in the sequence are connected.

This method is efficient as the number of Lambert arcs to compute, which often is the most computationally expensive part of the search, is known beforehand and scales linearly with additional bodies in the sequence. Also, filtering criteria can help to quickly narrow down the search space. It is important to note that having too tight criteria can cause the algorithm to skip over solutions that could have been feasible in optimization.

[2] JPL SSD Small-Body Database for Varuna: https://ssd.jpl.nasa.gov/tools/sbdb_lookup.html#/?sstr=2020000.

[3] 2047 JUGA to Varuna referenced from, "Solar System Escape Trajectories Using Outer Planetary Gravity Assists":https://trs.jpl.nasa.gov/handle/2014/53221.

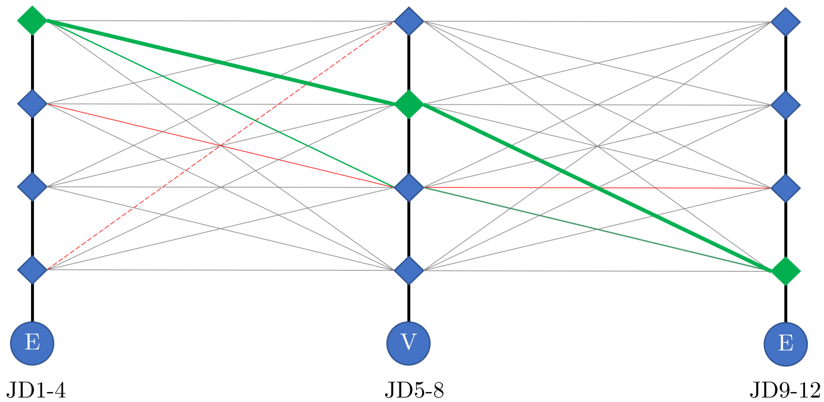

Figure 1: Broad trajectory search algorithm overview. Each diamond indicates a Julian date at the encounter body (Earth, Venus, Earth shown in this example). The dashed red line shows an infeasible transfer due to time (or failure to compute Lambert arc). The solid red lines connect the two Earth encounters, but the flyby V_∞ mismatch is too large or the altitude is too low. The solid green lines show a flyby trajectory that is acceptable.

2.1.2 $|V_\infty|$ Discontinuity Optimization

In order to have a valid trajectory, discontinuous V_∞ magnitudes between flybys must be minimized. Ideally, a completely ballistic flyby will have equal incoming and outgoing V_∞ magnitudes. The broad search results may get close to having valid flybys, but inherently guessing encounter epochs will yield discontinuous flybys that need to be altered. We can adjust the flyby encounter dates such that these discontinuities are minimized. To do this, the MATLAB Optimization Toolbox was utilized and a problem based optimization setup was configured. This utilizes MATLAB's FMINCON algorithm for direct-optimization subject to costs and both equality and inequality constraints. For this algorithm, linear constraints were not used, but non-linear equality and inequality ones helped to model the dynamics and flyby constraints. Equation (1) and (2) are the cost functions that were used to find an optimal solution. The first minimizes the required launch energy, and the second minimizes the total time of flight.

$$J_1 = min(C3) \tag{1}$$

$$J_2 = min(t_f - t_i) \tag{2}$$

The optimization algorithm is subject to the following non-linear equality constraint:

$$0 = \Delta\vec{V}_\infty \tag{3}$$

and non-linear inequality constraints:

$$0 \leq \vec{r_p} - (\vec{r}_{planet} + \vec{m}) \qquad 0 \geq C3 - C3_{max} \tag{4}$$

where $\Delta\vec{V}_\infty$ is a state vector containing all the V_∞ norm differences for each encounter. Similarly, $\vec{r}_p$ is the radii of each close approach to their respective body compiled into the vector $\vec{r}_{planet}$ with some minimum flyby altitude, once again compiled into the vector, $\vec{m}$. An additional constraint on the maximum launch C3 is also incorporated. The dynamics of the trajectory are propagated sequentially using Lambert arcs connecting the bodies given the optimization's iteration for the encounter dates. Because the sequence and encounters range of possible solutions are small, minimal differences were seen in using either cost function. Therefore, the minimum time cost function was used. Optimal solutions for a test 7 sequence trajectory were found within 5 seconds with feasibility, first-order optimality, and constraints criteria satisfied within tolerance. This meant $\Delta\vec{V}_\infty$ had components smaller than 1e-4 km/s. The algorithm's application to the current trajectory design is discussed in the following section.

2.1.3 Ephemeris Models

For all aspects of the trajectory design, having a reliable and fast computation method for planetary states is essential. For this design, two models were used: the Meeus algorithm, and the JPL Horizons Navigation and Ancillary Information Facility (NAIF) SPICE DE421 model. Meeus uses coefficient polynomials to approximate the planetary orbital elements and mean anomaly given a Julian centuries time value. It is important to note that despite being a fast means of computation, the Meeus algorithm will have deviations from the higher fidelity SPICE model. The SPICE ephemeris is the common standard for high fidelity trajectory design, and the planetary ephemerides model (DE421) can be loaded into MATLAB and GMAT for analysis. Though not the most current ephemerides model, DE421 is sufficient for this preliminary design investigation. The ephemeris model discussion is brought up due to the differences observed in the state of Uranus during the preliminary design. Figure 2 shows the variation between the Meeus algorithm and DE421 in the years 2029 to 2046. The Δ between the DE421 state and the Meeus algorithm are shown in the second and fourth sub-plots. From these, it is seen that the approximation of the Meeus algorithm appears to have a large mismatch in the X position coordinate (between 6e+7 and 7e+7 km) and has a large velocity mismatch of around 0.2 km/s in the X and Y components. These are measured with respect to the Ecliptic J2000 inertial frame. This discrepancy in the Meeus algorithm meant, that the B-Plane targets at Uranus would not be ideal for the TNO encounter. Because Varuna has an inclination of 17.22 degrees and the time between the UGA and Varuna is in excess of 2500 days, even small discrepancies in the B-Plane and energy targets at the UGA will result in very large deviations post-propagation. Therefore, in the preliminary design (before moving the solution to the higher fidelity model in GMAT), optimization was done using states queried from DE421. This is the same ephemeris model used by GMAT and ideally results in better B-Plane targets for the TNO encounter.

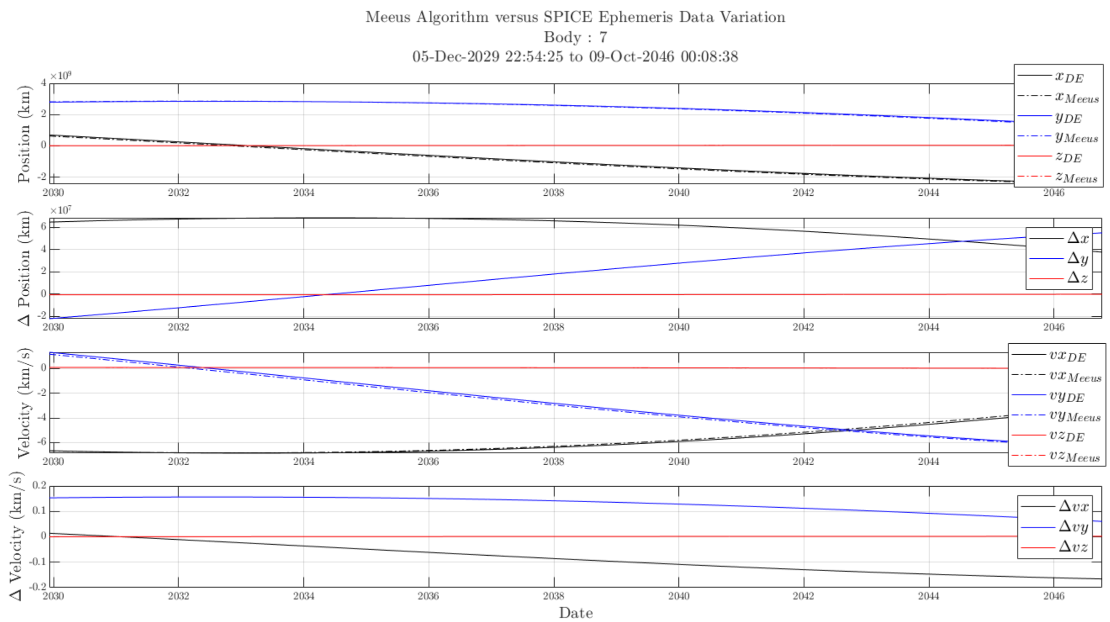

Figure 2: State of Uranus from 2029 to 2046 in the Meeus Algorithm versus DE421

Varuna is not in GMAT or DE421.bsp by default. Therefore, an extra ephemeris file '2020000.bsp' had to be loaded into MATLAB and GMAT for this leg of the trajectory. The .bsp file is from the JPL Horizons website which is custom generated for the date range valid for this mission.

2.2 General Mission Analysis Tool (GMAT)

NASA's GMAT software is a freeware, open-source, tool for mission design and trajectory optimization. The program has a graphical user interface and can also support scripting-level control using MATLAB or Python interfaces. GMAT is useful to optimize trajectories in higher fidelity dynamical models that can incorporate N-Body gravitational forces, solar radiation pressure, and atmospheric drag.

Propagators can be setup with different integrator choices, step size and tolerance criteria, and force models. The tool also allows for optimization of impulsive or finite-burn maneuvers with the capability of mass decrements. Along with the advanced dynamical models, GMAT offers users the ability to visualize the trajectory, create reference frames, and create data outputs such as tables and plots. Users design their mission in the "mission sequence" tab. Optimization is done by targeting which is frequently utilized with the following sub-criteria: 1) vary, 2) maneuver, 3) propagate, and 4) achieve. For example, when constructing a TCM, the TCM ΔV components can be varied such that when the maneuver is performed and the state is propagated downstream, certain flyby B-Plane targets and C3 energy can be achieved. While GMAT is powerful for optimization, it should not be used for sequence searches. GMAT is specialized to optimize trajectories in higher fidelity models and should have a good initial guess for each input parameter. The broad search and 2-body optimization from before serve as great starting points for this more advanced dynamical model trajectory optimization.

3 Trajectory Design

3.1 Optimality Criteria

A solar system escape trajectory is inherently energy maximizing. We want to begin with a low departure C3 to keep launch costs down and increase the payload to orbit, but we also want to ensure the spacecraft has enough energy to escape and reach the TNO. Therefore, each gravity assist was expected to be "low and fast" to continue boosting the heliocentric energy. A unique set of challenges come with this as conventional VEGA trajectories to the outer planets are usually energy minimizing at the final body to keep the capture ΔV costs low. However, there exists solutions for inner planetary flybys that can maximize the V_∞ into the Jovian gravity assist. These will likely have low flyby altitudes which would be of concern for EGAs which is discussed further on. As long as the launch C3 is small, significantly less than previous solar system escape spacecraft, the mass delivered could be high and thus extra weight for maneuver propellant can be included. Even if discontinuities exist in the solution, well-placed TCMs could make up for it which can keep the mass delivered to the interstellar medium fairly large. In summary, the key optimality criteria for this mission design are: 1) minimize the launch C3 to maximize mass, 2) have Earth flybys that are sufficiently high in altitude, and 3) reach the TNO in the shortest flight time possible which yields the fastest solar system escape speed.

3.2 Sequence Search Strategy and Optimization

The trajectory design process began with the broad search. This algorithm was initialized with the outer planetary flyby sequence from Earth and worked backwards to launch. A set of solutions were evaluated to find a transfer from Earth to Jupiter and onto Uranus. This solution was then extended to Earth again following a 2:1± or 3:1± Delta-V Earth Gravity Assist (DVEGA). Due to energy demands and planetary phasing, the 3:1+ solution was most fitting for this trajectory. This Earth-Earth flyby is not resonant and is designed to maximize the difference between the incoming heliocentric velocity vector and Earth's velocity. This in return increases the incoming and outgoing V_∞ magnitude. The greater the turn angle between these vectors, the greater the ΔV magnitude becomes through the flyby, but this will decrease the flyby altitude. The term "DVEGA" implies that a ΔV is required between the Earth flybys to achieve this energy increasing condition. This is usually in the form of a Deep Space Maneuver (DSM) performed at aphelion. For this mission, it was expected that this maneuver (or maneuvers) would have a larger cost than conventional TCMs. The trade off for a low launch C3 and higher payload, means we will need to perform this maneuver to maximize our post-EGA2 energy. Stepping backwards from this solution, a Venus flyby was added onto the sequence and finally the itinerary was completed with the Earth departure. This process took several broad searches to eventually find a sequence that met the criteria. A 10 day spacing was used for the inner planets and a 50 day spacing was used for the outer planets. The final broad search that was conducted had a spacing of 5 days between inner planets and 50 days for the outer bodies. This search consisted of 575 Lambert arcs (386 discarded solutions) and took 1.17 seconds to compute and plot on a 2015 Macbook Pro computer. Figure 3 shows its results. From the figure, it is seen that solution number 8 on the left hand sub-figure has the lowest sum of the flyby discontinuity V_∞ magnitudes. This trajectory is integrated using MATLAB's ODE45 and the 2-body equations of motion and is shown in the sub-figure

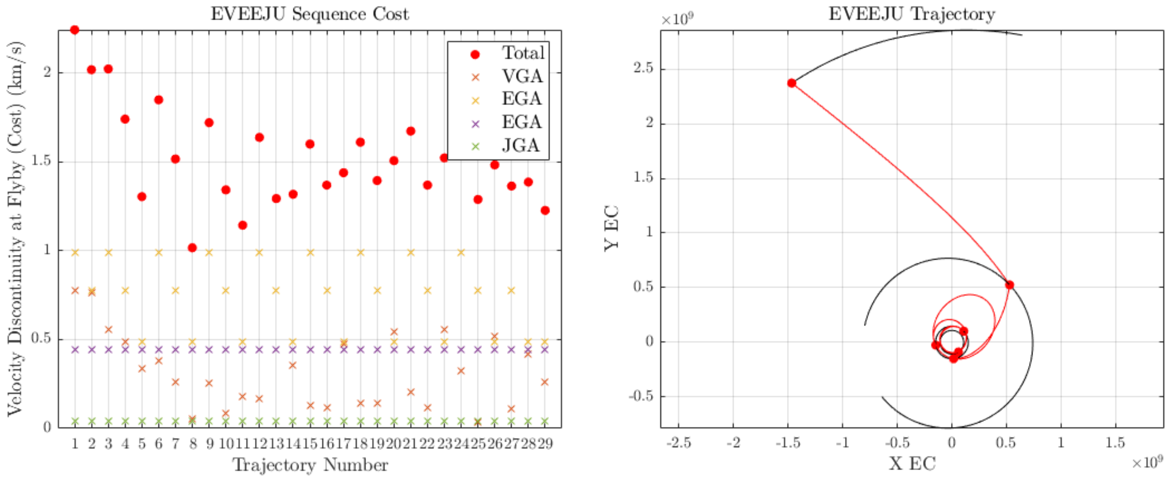

Figure 3: Broad trajectory search results of the EVEEJU sequence.

on the right. Notice that the EGA (shown has a yellow "x" in the left figure) has the highest V_∞ difference. This indicates that there is potentially an energy deficiency between the inner planetary sequence and the outer planetary portion. The sequence with the lowest cost was then used to start the optimization process.

Now that a good idea of the encounter dates is known, we can continue with the trajectory design process by optimizing the encounter dates to eliminate the discontinuities in ΔV_∞. The MATLAB Optimization Problem formulation was used to vary encounter dates to minimize these discontinuities subject to constraints of the flyby and launch energy. Before the design process, it was expected that flybys would be "low and fast", and this is evidently seen when optimizing the Earth flybys.

Enc.	V_∞ Mismatch (km/s)	
	$h_{min.}$=450 km	$h_{min.}$=350 km
VGA	0.000336988314380449	-2.21948131340355e-06
EGA1	0.080674422797317300	3.94900539735232e-05
EGA2	0.000274398724897651	-2.50999022632925e-05
JGA	0.000183130150361421	-6.32167999725652e-07
UGA	8.39192585111448e-05	4.39006555552623e-07

Table 2: V_∞ Mismatch from optimization for different minimum flyby altitudes of Earth.

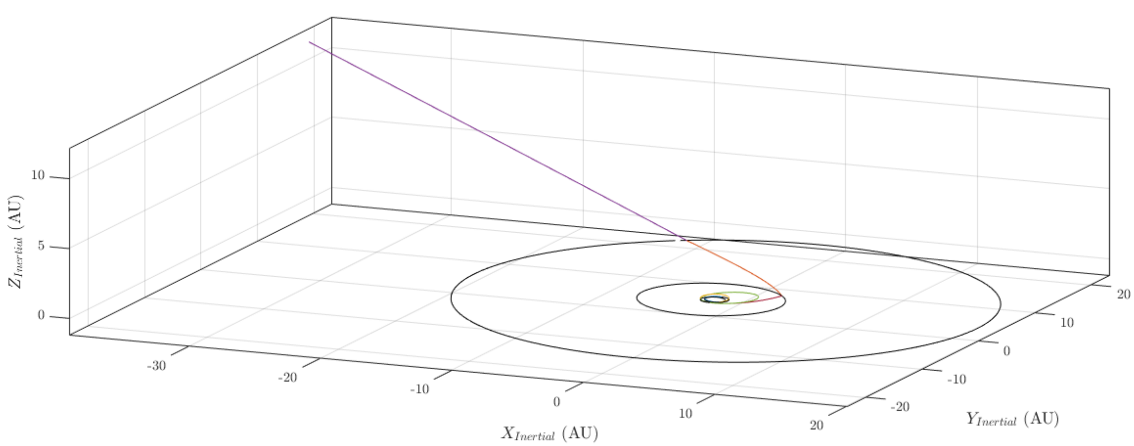

Figure 4: Optimized trajectory to Varuna in the 2-Body Model with DE421 and 2020000 SPICE data

Table 2 shows the post-optimization resulting discontinuities for two different close approach altitudes for the Earth flybys. It is evident that if the flyby altitude is lowered to 350 km, all the discontinuities become very small, and thus implying the trajectory is ballistic. However, a 350 km flyby altitude of Earth would require extended analysis in a detailed design on possible conjunctions and atmospheric affects. This is discussed a following section. Therefore, the decision to investigate a minimum of 450 km was taken into account despite the 80 m/s discontinuity at the first Earth flyby, despite being a powered flyby. To fix this, larger TCMs are to be expected to recuperate the energy loss through a higher flyby of Earth. This, and having a DSM, meant we can expect 2 TCMs between the Earth-Earth encounters to have fairly large maneuver costs. However, this is assumed to be okay as our optimality criteria of: 1) C3 minimization, 2) sufficiently high EGAs, and 3) getting to the TNO as soon as possible are all satisfied. Figure 4 is the integrated 2-body trajectory from Earth to Varuna with minimum EGA1/2 flyby altitudes being 450 km. Encounter dates, B-Plane coordinates, and energy at each flyby were extracted from this solution for GMAT. This solution has the EGA1 velocity discontinuity, but this will be accounted for with the use of TCMs and not a powered flyby.

4 GMAT Solution

4.1 Launch

Launch can be conducted from Florida at the Kennedy Space Center (KSC) and the Cape Canaveral Space Force Station (CCSFS). Alternatively, the Wallops Flight Facility (WFF) in Virginia can be utilized with a significant mass penalty. The expected launch declination angle is 22.759 degrees which is within the 40 degree limit for both launch facilities[4]. The required departure characteristic energy to encounter Venus is 15.79 km^2/s^2. At this C3, the spacecraft mass can be significantly increased compared to existing solar system escape spacecraft. Table 3 summarizes the Earth departure trajectory conditions including the required maneuver ΔV to put the spacecraft on the appropriate Earth escape trajectory. The right ascension angle (RLA) and declination angle (DLA) are computed with respect to the ecliptic as these quantities are derived from the outgoing V_∞ vector from Earth to Venus. A theoretical patched conics estimate for the Trans-Venus Injection (TVI) maneuver is provided for reference. This method uses the departure C3 to find the V_∞ and the required maneuver. The theoretical value is considerably close to the one computed by GMAT as the assumptions for the model hold well in this situation.

Earth Departure Conditions	
TVI Calendar Date	Dec 05, 2029 22:54:25.970
TVI Julian Date	2462476.45446725
Departure C3 (km^2/s^2)	15.7935231856157
Maneuver ΔV (km/s)	3.9004876939065
Patched Conics ΔV (km/s)	3.9004286515614
RLA (deg.)	-23.8159330098672
DLA (deg.)	22.7591416653568
$\lvert V_\infty \rvert$ (km/s)	3.97410658961428

Table 3: Earth Departure

The NASA Launch Services Performance Website[5] can give us an estimate into the mass of the spacecraft that can be delivered with existing launch vehicles. For the query, the SpaceX Falcon Heavy in the expendable and reusable configurations is considered. The ULA Vulcan VC6 is also a viable alternative. Both of these launch vehicles would depart from either KSC or CCSFS respectively. An alternative mission design would be to launch from WFF in Virginia using the Northrop Grumman Antares 232 vehicle. However, this vehicle would significantly reduce the mass delivered to orbit compared to the Falcon and Vulcan vehicles. Table 4 summarizes the mass delivered to orbit for these vehicles at the required departure C3. Figure 5 summarizes this information for a range of departure energies and the masses delivered. The SpaceX Falcon Heavy in the expendable configuration would

[4]NASA Interplanetary Mission Design Handbook:https://ntrs.nasa.gov/api/citations/20100037210/downloads/20100037210.pdf.

[5]NASA LSP Performance Website:https://elvperf.ksc.nasa.gov/Pages/Query.aspx.

Vehicle	Mass Delivered (kg)
Falcon Heavy (Expendable)	11020
Falcon Heavy (Recoverable)	4365
Vulcan (VC6)	8225
Antares (232)	1135

Table 4: Maximum Spacecraft Mass at C3=15.79 km^2/s^2

yield the highest launch mass for the spacecraft. Delivering more than 11,000 kg on an escape trajectory is a novel mission design that would enable science teams to carry many more instruments to study Uranus and the interstellar medium compared to existing missions.

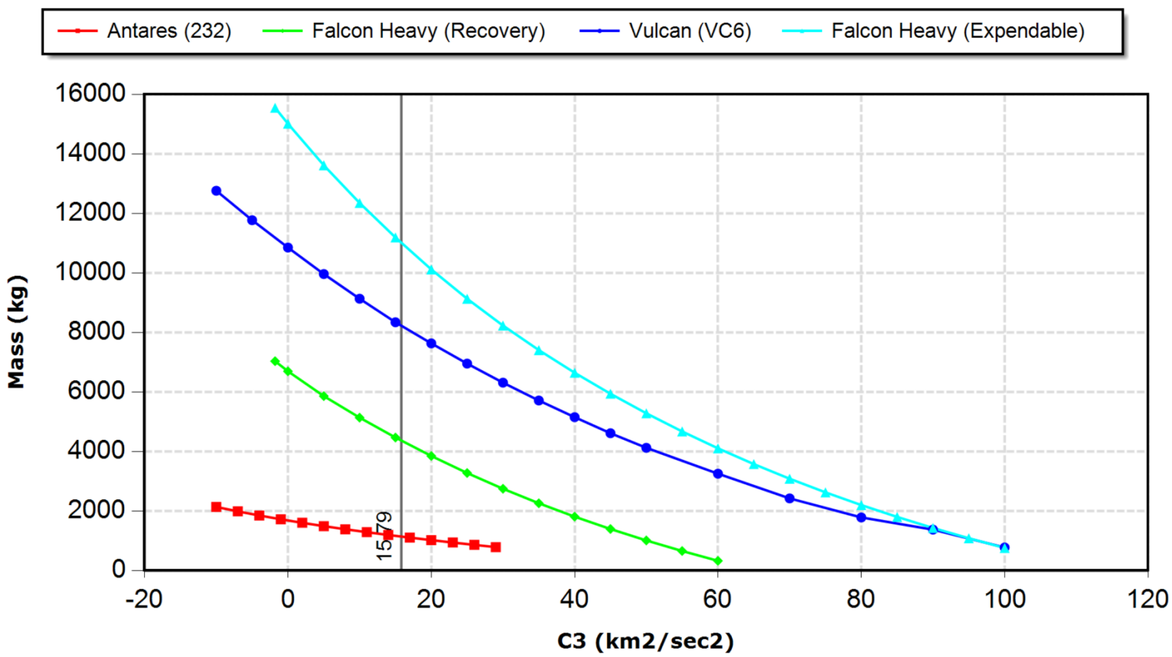

Figure 5: Launch Vehicle Spacecraft Mass Delivered at C3=15.79 km^2/s^2

4.2 Earth Flyby Considerations

This trajectory will come relatively close to Earth in at least one of the flybys. Table 5 includes the flybys of Earth, and it is seen that the spacecraft will get to 465 km altitude for the first encounter and 1485 km in the second. Both flybys would require conjunction analysis and EGA1 will require maneuver biasing to keep the risk of impact minimal. A solar system escape type mission is likely going to utilize radio thermoelectric-isotope generators (RTGs) which contain nuclear substances. A potential impact with the Earth could mean this can be scattered through the atmosphere, and would require additional approvals and analysis (similar to the Cassini mission). The following considerations would be analyzed in a detailed design of the trajectory. To satisfy the optimality criteria of this preliminary design, the EGA1 flyby altitude was raised from 350 km to 465 km. This serves as a precaution from flying to close to Earth which reduces the adverse affects due to the upper atmosphere. Because large maneuvers do not occur before this flyby, it is likely that maneuver execution errors could be small. The post EGA1 86 m/s maneuver (TCM4) at the heliocentric periapsis and apoapsis maneuver (TCM5) of 36 m/s are essential to make up for the lost energy gain from EGA1.

4.3 Mission Design

4.3.1 Planetary and TNO Encounters

Table 5 summarizes all the encounters with their flyby energy (C3), radius, altitude, and B-Plane coordinates. Table 6 summarize the incoming and outgoing V_∞ vectors for each encounter presented in the ecliptic J2000 inertial frame. Several key takeaways from the encounters include the total mission time, Earth flyby altitudes, and the possibility for science at Uranus due to the close approach altitude. The total mission time is under 17 years and the trajectory takes less than 10 years to reach Uranus. While slower than Voyager, this mission does benefit from a large payload capability. Next, the Earth flyby altitudes are both above 450 km. The implications of these flybys are discussed in the sub-section above and further on in the maneuvers summary section. The possibility for science at Uranus is also interesting to point out due to the low close approach altitude. If this altitude is too low, additional maneuvers can be planned in detail design to specifically target Uranus, and compromise on the TNO close approach, to meet the decadal survey goals. The Varuna close approach is within 100,000 km. Due to the long propagation period of over 2500 days from the UGA, any errors in the flyby of Uranus or maneuvers have an amplified affect downstream. Ideally, the radius of close approach to Varuna can be reduced, but the maneuvers are very small and hard to converge on. For this preliminary study, a 91,700 km approach to the TNO was considered sufficient.

Enc.	Date	C3 (km^2/s^2)	Flyby Radius (km)	Flyby Alt. (km)	BR (km)	BT (km)
TVI	05 Dec 2029 22:57:24.281	15.7935	-	-	-	-
VGA	19 May 2030 10:22:11.002	48.0781	11487.74602	5435.9460	8081.9929	14896.0514
EGA1	06 Apr 2031 15:41:42.857	114.8170	6843.430131	465.2938	2023.3149	-9500.2222
EGA2	28 Jun 2034 01:58:10.502	121.0165	7863.628457	1485.4921	1170.6846	10595.6633
JGA	23 Oct 2035 10:07:06.686	139.2994	326436.8373	254944.8373	5664.6333	836905.4508
UGA	06 Nov 2039 12:37:35.267	113.0306	28440.61668	2881.6166	24787.668	34509.9084
Varuna	11 Oct 2046 12:33:36.052	407.5873	91674.20974	90774.2097	-	-

Table 5: Encounters Summary

Encounter	V_∞ Vector (km/s) V_x	V_y	V_z
Earth	3.35262275029314	-1.47979662468253	1.53741530804488
VGA	5.99220354709019	-3.4756120257708	-0.272071185072071
	6.52661640281043	1.06278622712841	2.08298854941178
EGA1	9.85166368886271	3.72650267451788	-1.81046924617297
	10.2583366151464	-3.27373074208382	0.00433181157242232
EGA2	6.71129500045267	-8.71533665352276	0.000468829327699497
	10.4012658942387	-3.45389738450242	-0.9427996889466
JGA	11.425873491935	3.26570396913663	-0.251524398851049
	-4.15604434309761	11.1355376348165	0.100610248191638
UGA	-8.67271924416218	16.014669877196	0.254536654782811
	-15.2643630827232	6.33925300767765	7.65288489896048
Varuna	-18.2550565925485	5.42238274052348	6.78980240699023

Table 6: V_∞ vectors for each encounter from the $h_{min.}$=450 km optimization. The top row of each encounter indicates the incoming velocity, and the bottom row is the outgoing.

4.3.2 Trajectory Correction Maneuvers (TCMs)

Due to the higher order gravitational model, and discontinuities in velocity through encounters, the trajectory requires correction maneuvers throughout to minimize errors. TCMs were generally spaced within 15-50 days before or after each encounter, and the strategy used was to either target the current flyby's: B-Plane, Modified Julian Date, or C3, or to target these properties of the next encounter. Targeting the next flyby's states prior to the current encounter yields a considerable cost savings downstream by leveraging the energy gain through the immediate flyby. Maneuvers were not added close to the flyby (under 15 days) due to mission design considerations such as execution errors, orbit determination, and preventing a nearly powered flyby solution. Trial and error for maneuver placement and target selection yielded lower maneuver costs. Table 7 summarizes the Trans-Venus Injection (TVI) all the trajectory correction maneuvers (TCMs) used for this mission. Note that maneuver execution errors are not modeled in this mission scenario. TCM1 is executed within 20 days from launch to target the B-Plane components and encounter date. This maneuver had a sufficient delivery to Venus and so the second TCM was used to target the first EGA's encounter date and energy. This modified the Venus flyby slightly to achieve the conditions at EGA1. TCM3 was used to hone in on the B-Plane components and encounter date of EGA1. this was essential to ensure that the Earth-Earth leg delivery to EGA2 was as precise as possible. Recall that a DSM is necessary for the Earth-Earth transfer, and that there was an 80 m/s discontinuity at EGA1 in optimization. Because of these anomalies, TCM4 and TCM5 were larger than usual to make up for the energy loss through EGA1 and to encounter Earth at the appropriate state for EGA2. EGA2 has another TCM for accurate time, energy, and B-Plane targets. Enroute to Jupiter, TCM7 and 8 are performed to target the flyby conditions. TCM9 occurs before the JGA to target the UGA, and is abnormally high. This is likely due to a frames issue and will be investigated in detail design. Despite being a large maneuver, TCM9 is still within the 100 m/s threshold for the scope of this project. TCM10 corrects flyby errors from JGA and ensure an accurate delivery to the UGA. Finally TCM11 and TCM12 occur before the UGA and after it to ensure a reasonable delivery to Varuna. As mentioned in the section above, the maneuvers targeting the TNO flyby are very small and difficult to converge on due to the long propagation time from UGA to the Varuna close approach. Additional ΔV for the maneuver budget should be considered to target Varuna to flyby at a desired flyby altitude. For this preliminary design, reaching under 100,000 km to the body was used to satisfy the close approach condition. All together, 12 TCMs were designed with a total ΔV sum of 293 m/s. Including the TVI burn, the total is increased to 4.19 km/s.

Maneuver	Calendar Date	Julian Date	ΔV (m/s)	Targets
TVI	05 Dec 2029 22:57:24.281	2462476.45653103	3900.4876939065	C3, RLA, DLA
TCM1	21 Dec 2029 10:57:24.281	2462491.95653103	20.88613272183885	BR, BT, JD
TCM2	29 Apr 2030 10:22:11.002	2462620.93207178	12.40702610957967	JD, C3
TCM3	18 Sep 2030 15:40:25.355	2462763.15307124	10.24904270853394	BR, BT, JD
TCM4	18 Apr 2031 15:41:42.857	2462975.15396825	86.15932065275253	C3, RMAG
TCM5	25 Dec 2032 00:18:00.004	2463591.51250005	36.45211861059578	BR, BT, JD, C3
TCM6	04 May 2034 02:11:39.586	2464086.59143039	2.077614703550245	BR, BT, JD, C3
TCM7	08 Jun 2034 01:58:10.502	2464121.58206600	12.00265596592812	BR, BT, C3
TCM8	02 Aug 2034 12:50:23.342	2464177.03499238	3.061709835028656	BR, BT, C3
TCM9	20 Nov 2034 13:05:04.475	2464287.04519068	90.09247027326747	BR, BT, JD, C3
TCM10	29 Jul 2039 12:38:24.727	2465999.02667508	8.252365711698732	BR, BT
TCM11	07 Oct 2039 12:37:35.267	2466069.02610263	0.04001130252587617	RMAG
TCM12	20 Apr 2041 13:04:33.851	2466630.04483624	11.31500012870081	RMAG
$\sum$ TCM ΔV			292.995469	
$\sum$ ΔV			4193.483163	

Table 7: Maneuvers Summary

4.3.3 Comparison to a Hohmann Transfer

A Hohmann transfer from 1 AU to 42.718 AU is inefficient. Varuna also has a 17.2 degree inclination, making this approximation not accurate. However, if this transfer were to be considered, the departure $|V_\infty|$ from Earth would be 11.8527 km/s and the transfer time would take 18,664 days. The required maneuver from a 300 km Earth parking orbit is 8.3944 km/s. For comparison, this current trajectory's $|V_\infty|$ 3.9737 km/s, reaches Varuna in 2527 days, and has a departure maneuver of 3.9005 km/s.

4.3.4 Extended Mission to the Interstellar Medium

The Varuna encounter occurs at a V_∞ magnitude of 20.188 km/s, and the heliocentric C3 of the escape trajectory is 420.16 km^2/s^2. This means the spacecraft will escape the solar system at 4.32392 AU/year in the direction of the Leo and Virgo constellation. Additional mission objectives for heliopause and interstellar science can be continued as an extended mission. Due to the large mass delivered, the spacecraft can either have more RTGs for power, or more science instruments onboard to conduct detailed data collection. Previous probe missions have been bounded by stringent mass requirements, making the instrument selection and data collection constrained. This mission does not have this constraint to the same degree and so more flexibility in payloads and science can be incorporated.

5 Conclusions

A preliminary trajectory design to the TNO Varuna is created and implemented in GMAT. The mission departs Earth in December 2029 with a C3 of 15.79 km^2/s^2 and can carry up to 11,000 kg of mass onboard the SpaceX Falcon Heavy Expendable. The trajectory requires gravity assists of Venus, Earth twice, and Jupiter to reach Uranus in under 10 years. To appeal to the goals of the 2022 Decadal Survey, science can be conducted through the Uranus flyby. In under 17 years of the mission, the spacecraft will flyby the TNO Varuna before escaping the solar system at 4.32 AU/year. The optimality criteria for this mission are satisfied, and the chance for an extended mission to the interstellar medium is possible. This mission is a novel trajectory designed carry a massive spacecraft to study Uranus, Varuna, and beyond.

5.1 GMAT Mission Graphics

The following figures are taken from the GMAT simulation graphics window. Note, the Varuna encounter figure not presented as model for asteroid is not present in GMAT simulation.

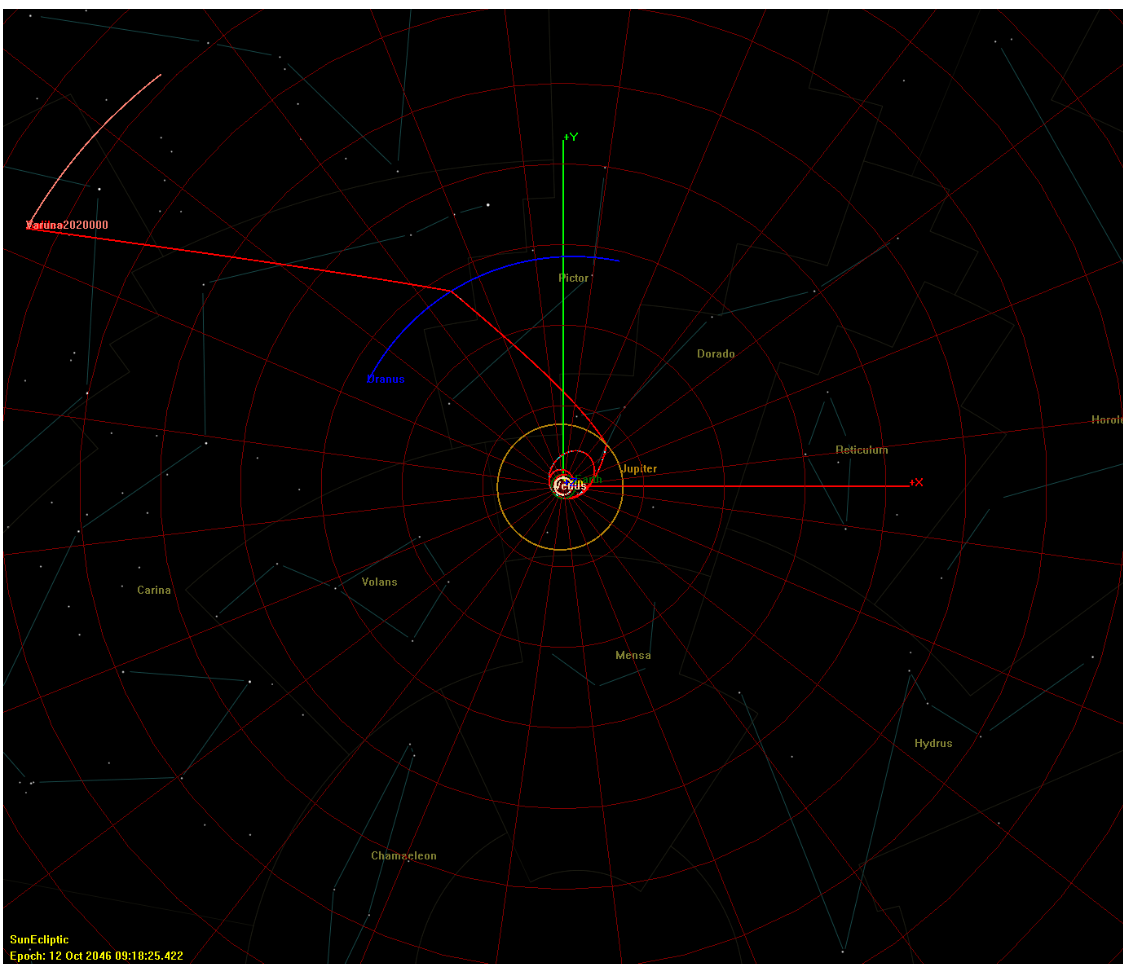

Figure 6: Top-Down Heliocentric View of Entire Trajectory

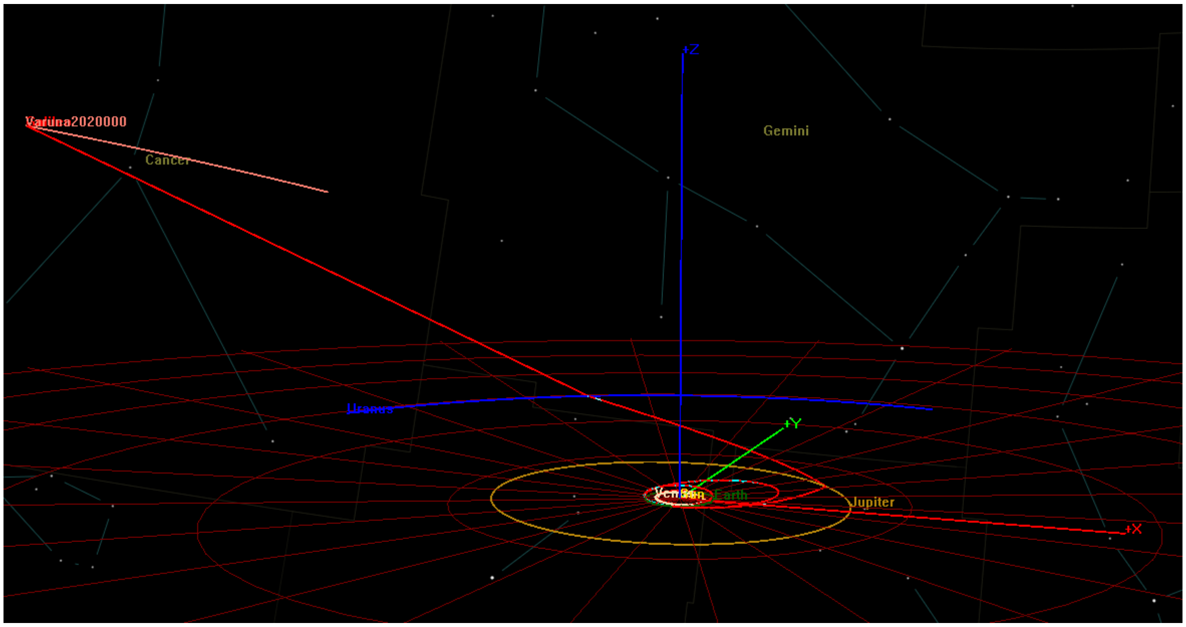

Figure 7: Heliocentric View of Trajectory Showing Varuna's Inclination w.r.t Ecliptic

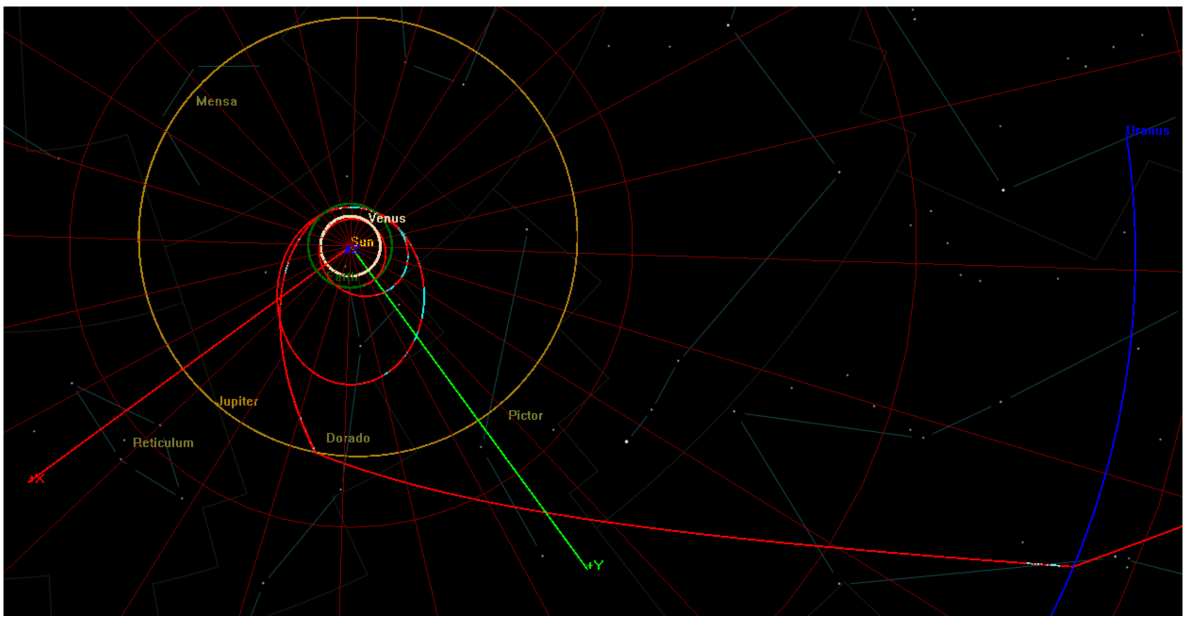

Figure 8: Earth-Uranus Portion of the Trajectory

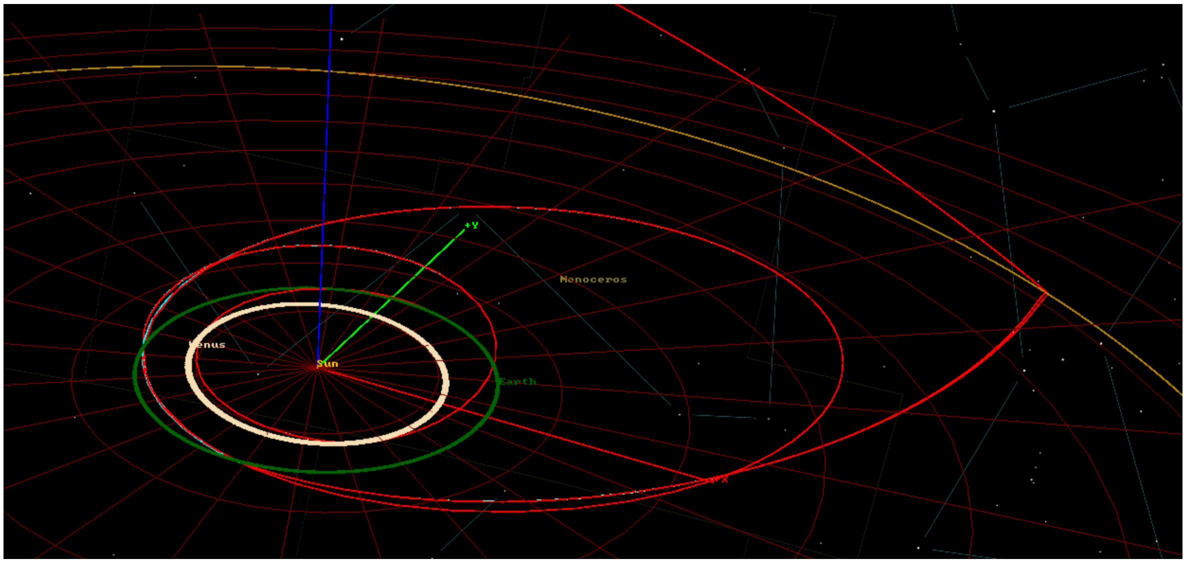

Figure 9: Tilted View of the Earth-Jupiter Portion

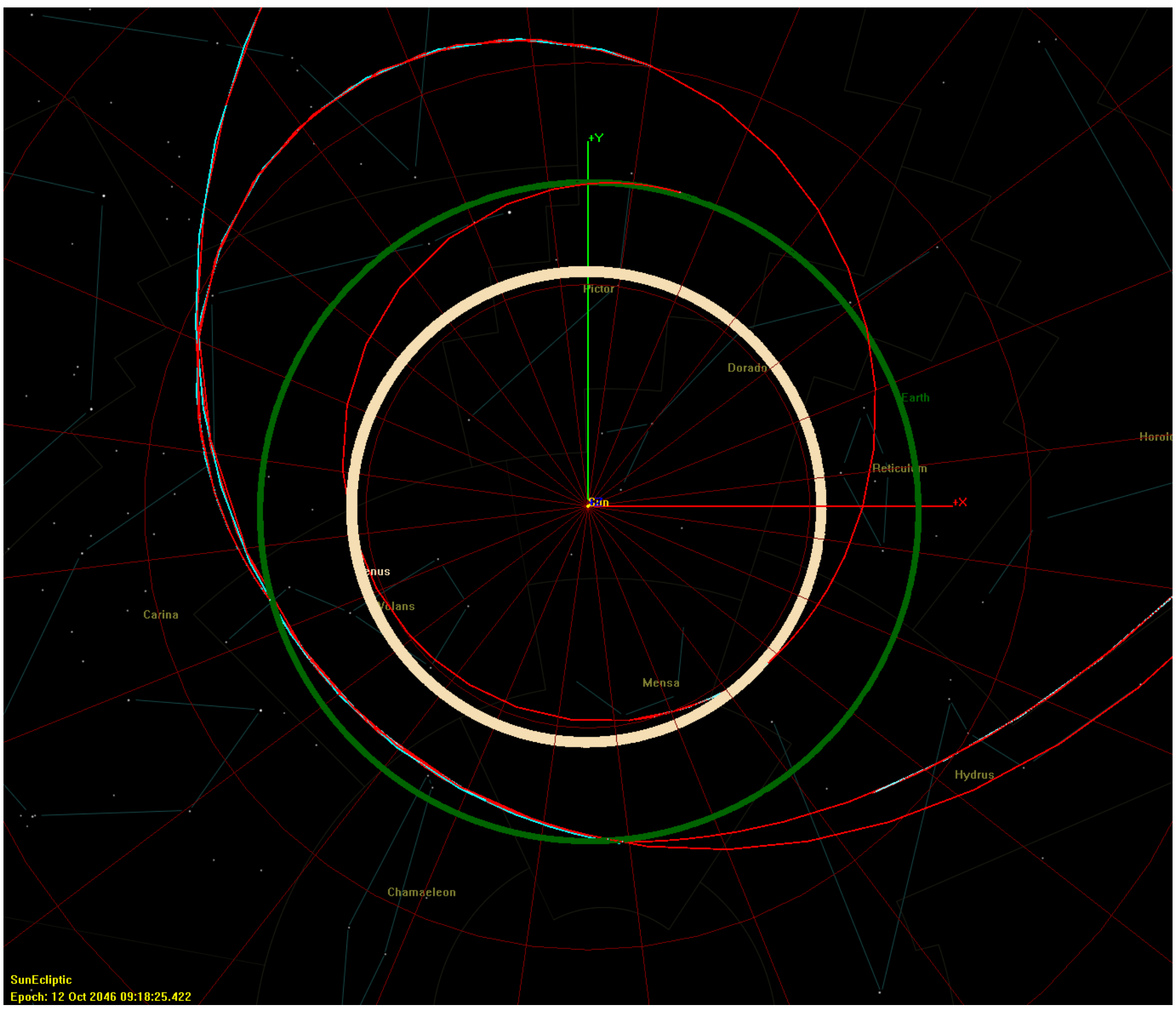

Figure 10: Inner-Planets Portion of the Trajectory

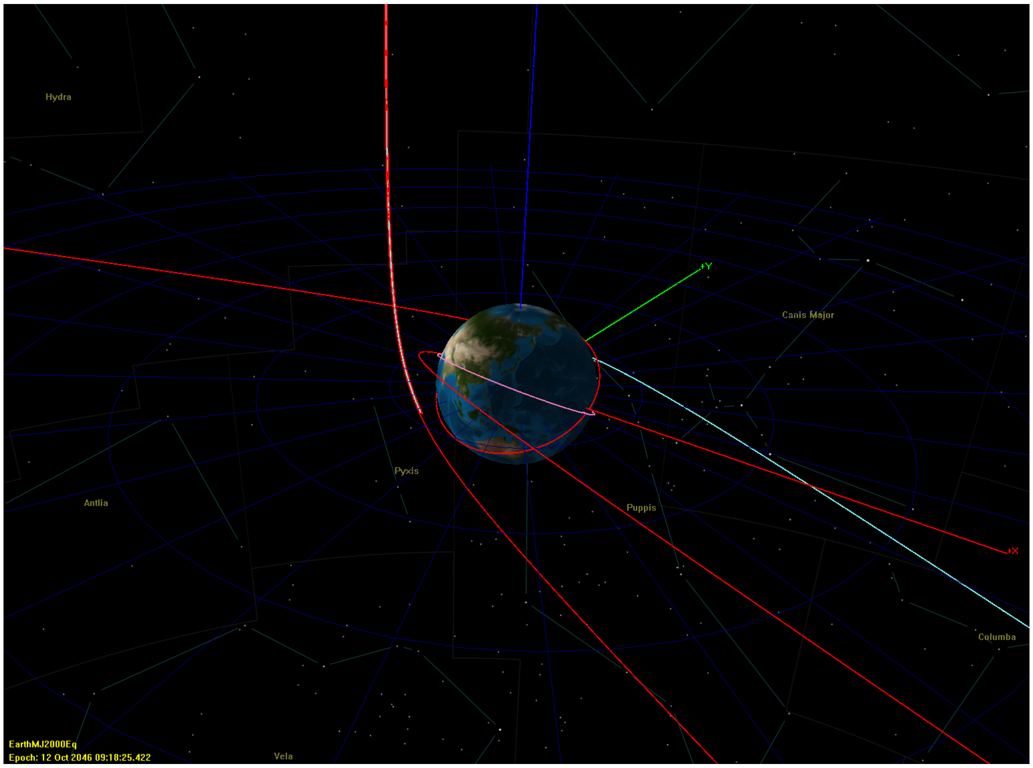

Figure 11: Earth Launch, EGA1, EGA2, and flyby targeting propagations

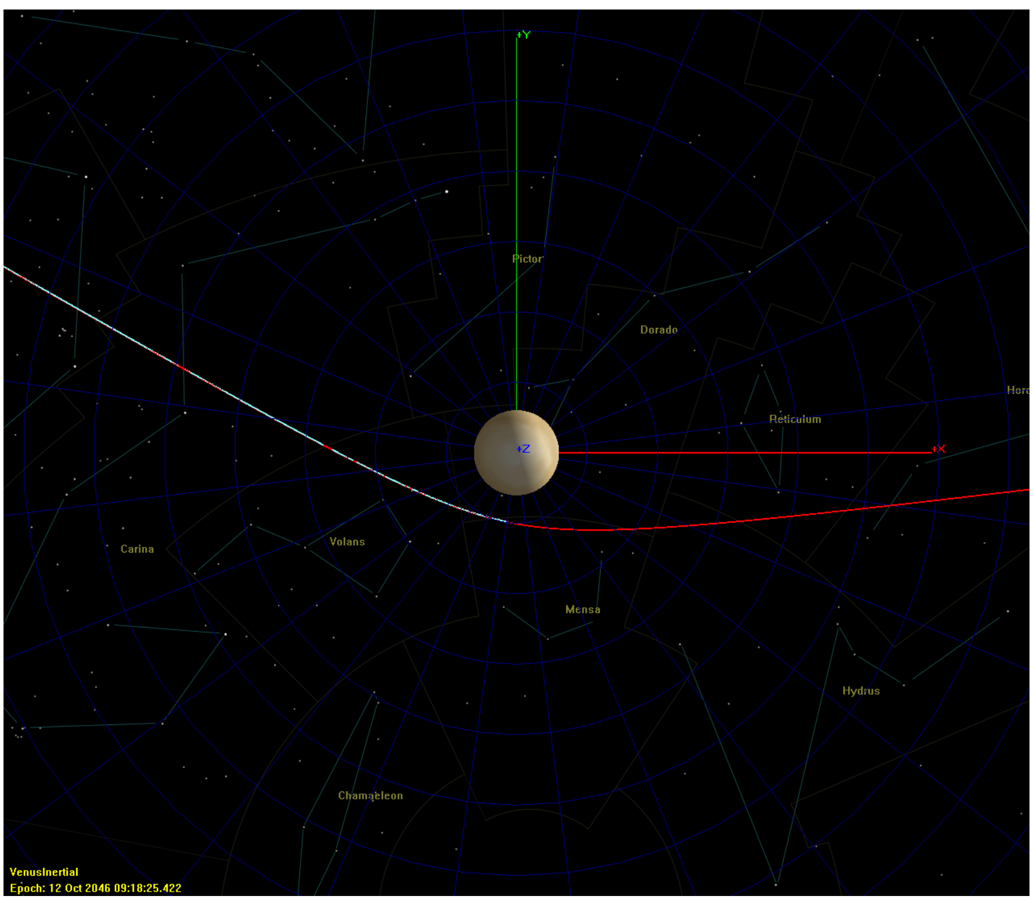

Figure 12: Venus Flyby Trajectory

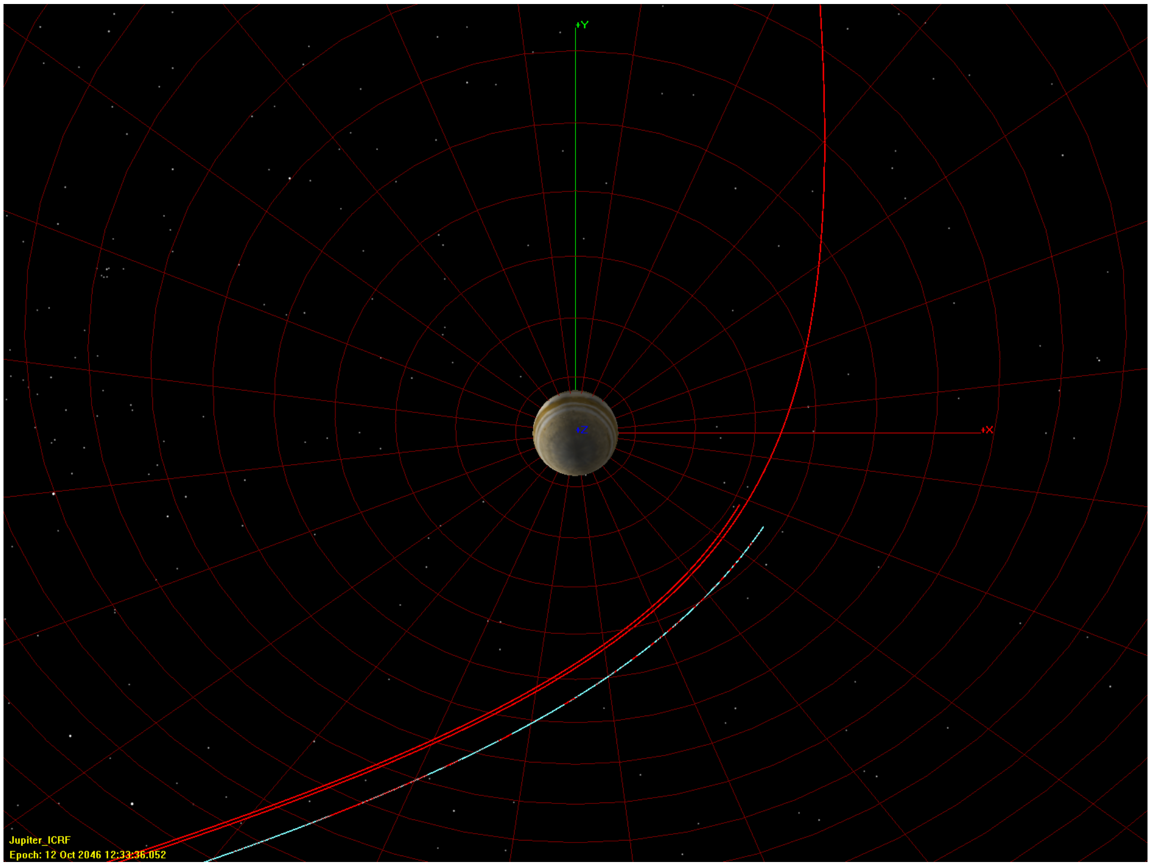

Figure 13: Jovian Flyby Trajectory

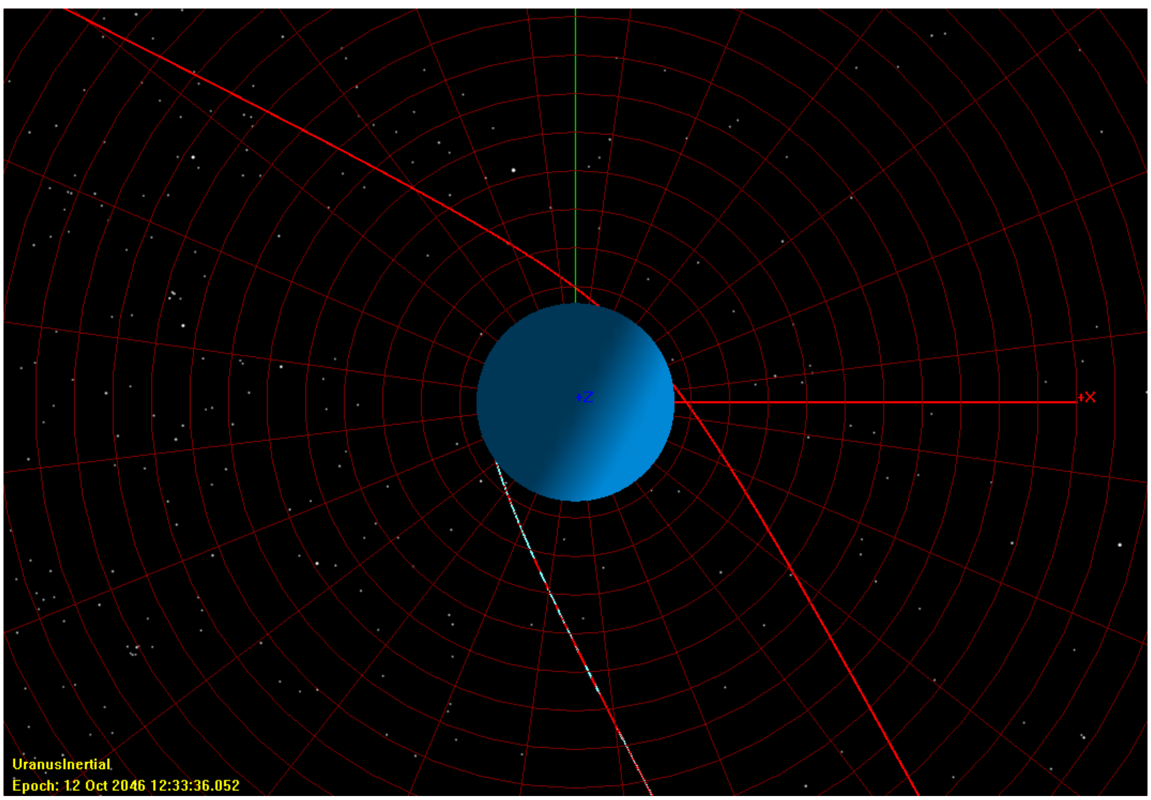

Figure 14: Uranus Flyby Trajectory

6 Appendix A: Porkchop Plots

For the trajectory search, Porkchop plots (PCP) were not used, but PCPs are included for reference. The trajectory's selected encounter dates are shown on each plot with a star and crosshairs.

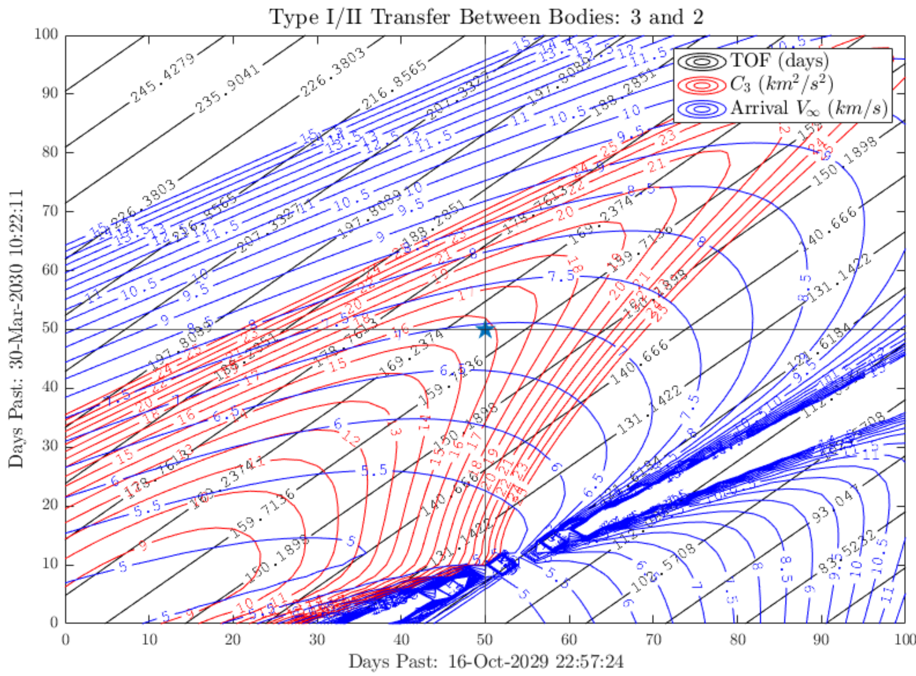

Figure 15: Earth to Venus PCP of the departure C3 (km^2/s^2), TOF $(days)$, and arrival $|V_\infty|$ (km/s)

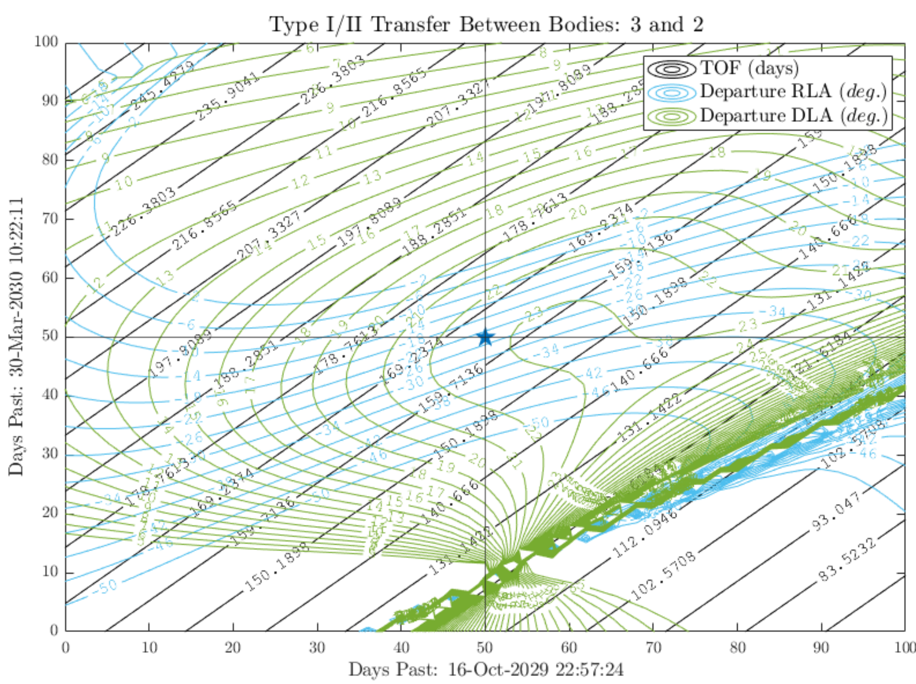

Figure 16: Earth Departure to Venus PCP of the TOF $(days)$, RLA (deg), and DLA (deg)

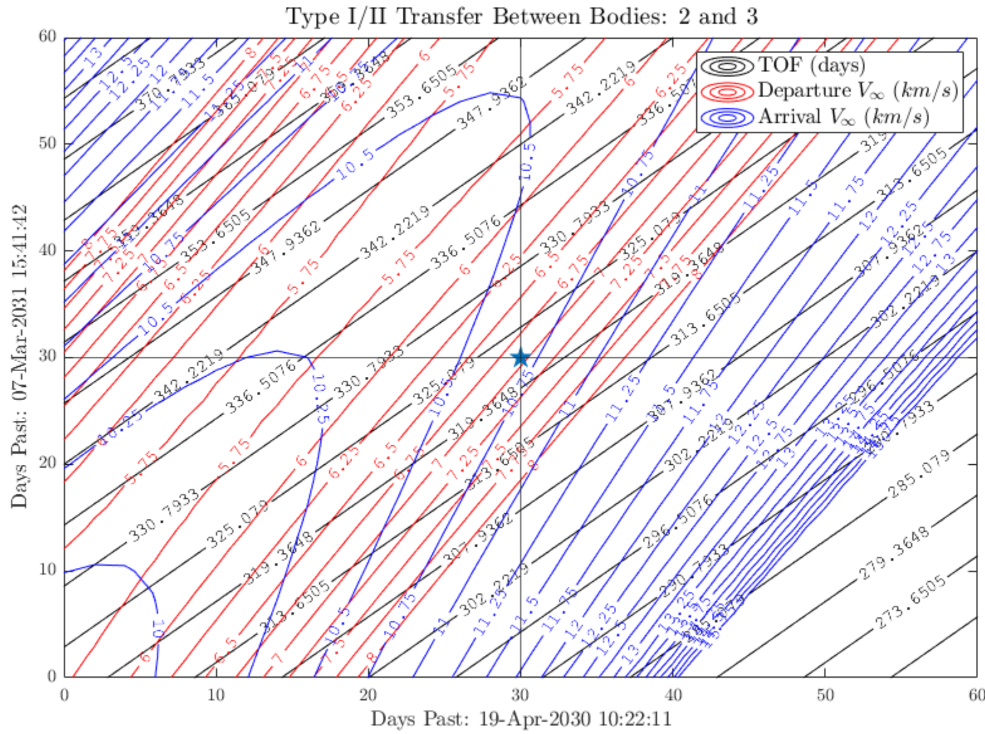

Figure 17: Venus to Earth PCP of the TOF $(days)$, departure, and arrival $|V_\infty|$ (km/s)

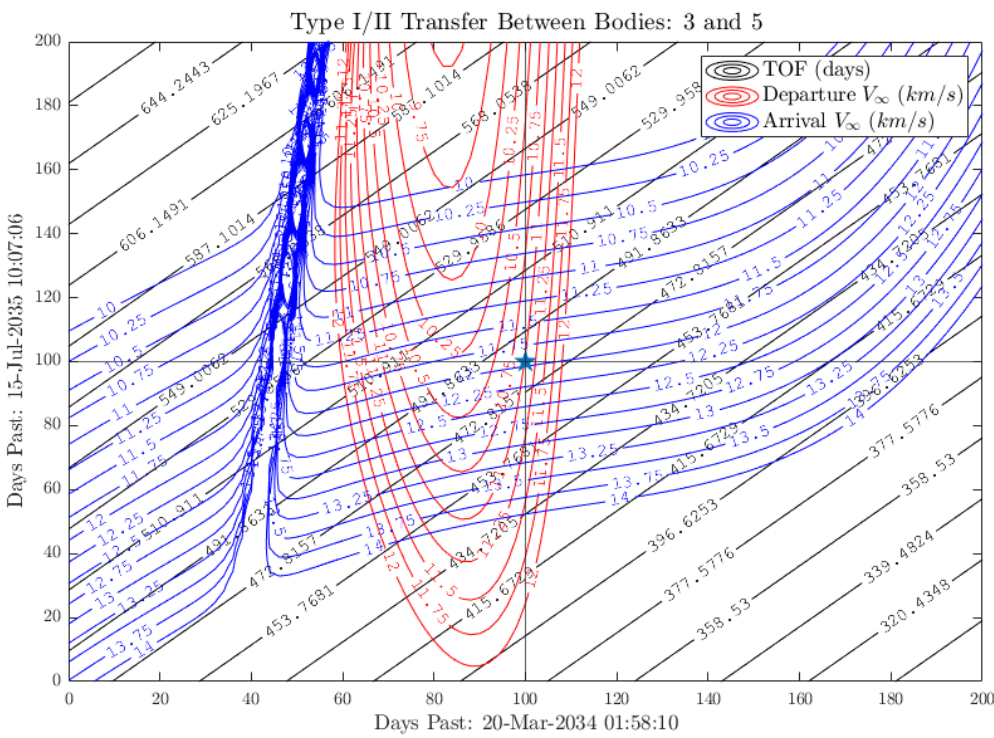

Figure 18: Earth to Jupiter PCP of the TOF $(days)$, departure, and arrival $|V_\infty|$ (km/s)

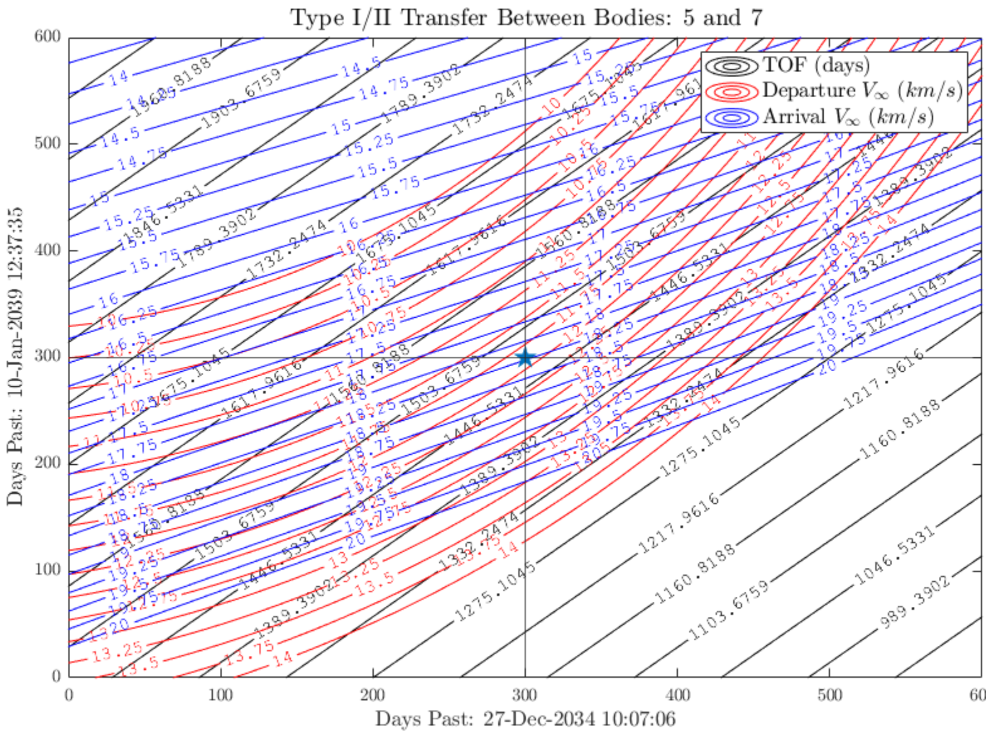

Figure 19: Jupiter to Uranus PCP of the TOF $(days)$, departure, and arrival $|V_\infty|$ (km/s)

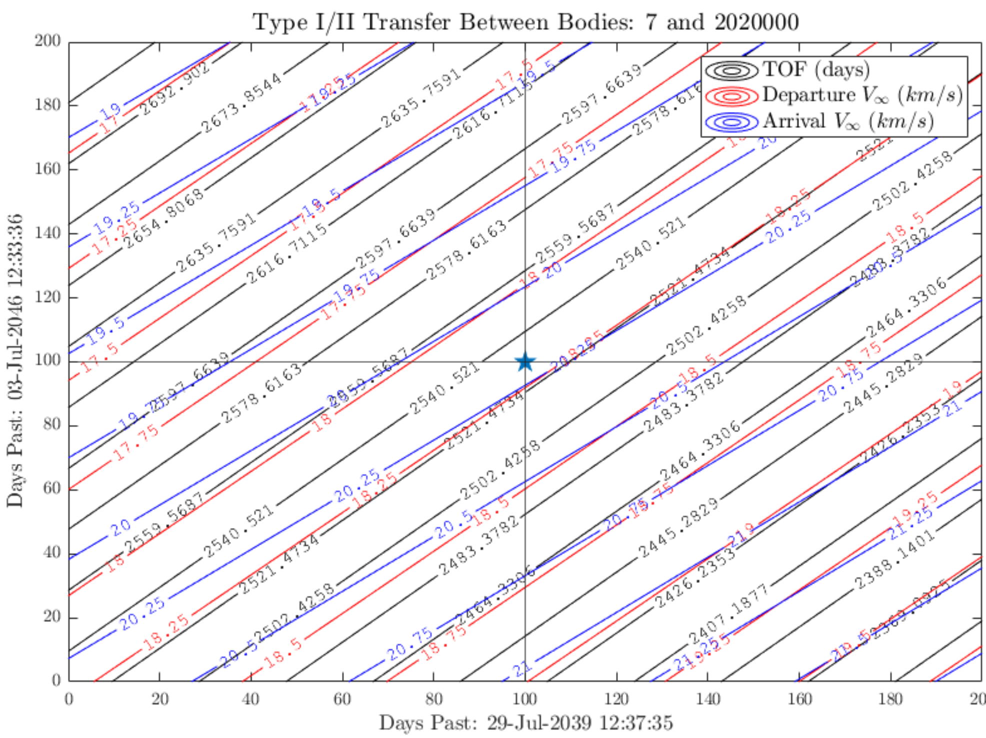

Figure 20: Uranus to Varuna PCP of the TOF $(days)$, departure, and arrival $|V_\infty|$ (km/s)